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**A REVIEW OF INSTABILITY AND NOISE PROPAGATION
IN SUPERSONIC FLOWS**

By

Q. Isa Daudpota, Research Associate

Principal Investigator: William D. Lakin

Final Report
For the period ended May 31, 1990

Prepared for
National Aeronautics and Space Administration
Langley Research Center
Hampton, Virginia 23665

Under
Research Grant NAG-1-881
Dr. John M. Seiner, Technical Monitor
ACOD-Aeroacoustics Branch

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INTRODUCTION

~~The~~ original purpose of this project was to develop analytical and numerical models ^{WERE TO BE DEVELOPED} for noise production in supersonic jets, wakes and free shear layers. While the effort was concentrated initially on these aspects, other topics were also pursued — most of these were of direct interest to the Jet Noise Group of the Aeroacoustics Branch. ^{IS GIVEN} This final report will give an overview of subjects reviewed and the investigations that were carried out.

A significant effort was devoted to numerically predicting the flow field of a turbulent supersonic wall jet. This information is necessary for computing the pressure in the far field. A CFD code developed by Dash and co-workers (1986) was used to obtain the mean flow.

The wall jet was selected because it represents a generic flow that can be associated with plug nozzle in supersonic engines. It combines the characteristic of a boundary layer with that of a free shear flow.

The spatially evolving flow obtained using Dash's code would form the input for the stability analysis program. This analysis would determine the large scale instability wave within the flow. The far field pressure can be computed from the shape of the evolving large scale structure by asymptotic methods. →

The method of Tam and Morris (1980), which involves the linear stability analysis of a mean velocity profile, can be used to evaluate the wall jet noise field. This approach considers small perturbations imposed on the mean flow which remains unaltered by the growth of the perturbation. In practice, however, it is noticed that small perturbations in flow quantities such as pressure at the lip of a jet can dramatically alter the undisturbed mean flow profile. This important

sensitivity of the mean flow to the perturbation cannot be taken into account by the formalism of Tam and Morris, and hence an approach initially developed by Malkus (1956) was considered worth investigating. Through this method, an $O(\epsilon)$ perturbation can be allowed to have an $O(1)$ effect on the mean flow. The procedure will be described later.

There has been a renewed interest in the linear and nonlinear stability theories of compressible flows. Most of the original effort in this field is summarized in Mack (1984). This work relates principally to planar 2-d and 3-d flows such as those over flat plates and in free shear layers. It has been known for sometime that transverse curvature of streamlines can have a significant effect on the stability characteristics. To elucidate this effect, an axisymmetric flow over the surface of a cylinder and also along the surface of a cone were studied, Macaraeg & Daudpota (1990). Findings from this paper will be summarized in one of the following sections.

→ Finally, flow characteristics obtained from a program that analyses the turbulent downstream supersonic flow in a nozzle are described and compared with experimental results. S N P

The following sections will deal with:

- (a) Noise emission from supersonic wall jets
- (b) An asymptotic method for computing far field noise from supersonic free shear layers
- (c) Linear stability of supersonic flows with transverse curvature
- (d) Boundary layer effect in supersonic nozzle flows.

(a) Wall Jet

Wall jets are of great engineering importance. Their diverse applications range from industrial heating, cooling, and ventilation to the field of advanced

airfoil design. The present research project has been primarily concerned with an analysis of the noise generation from supersonic wall bounded jets.

As a theoretical problem, subsonic wall jets have attracted considerable interest. This is mainly because the configuration of a free jet interacting with a boundary layer presents many challenges for the testing of computational turbulence models against experimental findings. A comprehensive review is given by Launder and Rodi (1983). Very little, if any, research has been done on supersonic wall jets, where the flow is further complicated by the presence of shocks. The presence of shocks and of the wall add greatly to the generation of noise. From a practical design point of view, the supersonic wall jet problem has applications to some of the proposed aircraft in which the propulsion system is integrated with the fuselage.

One formulation for analyzing a supersonic free shear layer and the noise field it generates in the far field has been developed by Tam and Morris (1980). A self similar velocity profile for the developing shear layer is assumed. The method of multiple scales is then used, supplemented by matched asymptotic expansions to derive results for the far field noise. This approach is valid for subsonically convecting disturbances, and has been extended to supersonically convecting disturbances by Tam and Burton (1984).

In the current project, mean velocity profiles for the wall bounded jet were generated numerically rather than choosing an assumed profile. Only after this calculation is completed would a stability analysis be carried out. For this analysis it was intended that the method developed by Lakin and Reid (1982) for boundary layer flows would be extended to the wall jet configuration. This method has the advantage of yielding uniformly valid asymptotic solutions over the entire region of interest, and it provides a consistent approach to matching the near and

far fields. It should be helpful in avoiding some of the technical problems inherent in the method of multiple scales.

At the start of the present study, an effort was made to use an existing code to generate the required mean velocity profiles. In particular, the code SCPVIS, developed by Dash and Wolf (1984), was tried. This code has been successfully used for various 2-D flows at Langley, and its documentation suggested that it was possible to adapt SCPVIS for a wall bounded jet. However, this proved not to be the case. After experimentation, followed by a discussion with Dash, it was realized that this code cannot be modified to treat the wall boundary condition in a satisfactory manner. A newer code, specifically developed for wall jets WLJET (Dash et. al, 1986) was made available by Dash, and this code was expected to yield the mean velocity profiles that are required for the stability analysis.

The program WJET solves the higher order curved boundary layer equations. The equations are cast in surface-oriented s, n coordinates and include a tracer species equation for ϕ ($\phi=1$ in unmixed jet, $= 0$ in airstream) to delineate the jet/air mixing region. A classical Boussinesq approximation is utilized to relate turbulent stress terms to mean flow gradients, with the parabolized stress terms retained. Turbulence closure is achieved using the two-equation $k\epsilon$ model with standard coefficients.

The approach taken in WJET involves combining:

- (1) a parabolic solution of the streamwise momentum, energy, species parameter, and turbulence model equations with the streamwise pressure gradient term, $\partial P/\partial s$ (s, n) specified - this solution yields the variation of U , H , ϕ , k and ϵ :
- (2) an elliptic/pressure-split solution of the coupled continuity and normal momentum equations in subsonic regions which yields the variation of pressure and normal velocity across the wall jet; and,

(3) a hyperbolic/upwind characteristic-based solution of the coupled continuity and normal momentum equations in supersonic regions which yields the local pressure and flow angle.

These three solution procedures are unified in the WJET code to provide generalized spatial marching capabilities for a broad category of wall jet problems.

The WJET parabolic algorithm integrates the U momentum, H , ϕ , k and ϵ equations in mapped rectangular coordinates. The mapped, vectorized equations are spatially integrated using an upwind/implicit algorithm. A fixed number of grid points are distributed between the wall ($\eta=0$) and the outer viscous boundary ($\eta=1$) whose growth is obtained via adaptive methodology keyed to the edge gradients. The distribution of grid points, $\eta(I)$, remains invariant throughout the calculation and the stretching utilized can be arbitrarily stipulated, or specified using built in grid distribution parameters. The equations are solved in an uncoupled manner (the source terms are solved explicitly) and the difference equations then take standard tridiagonal form.

Fig. (1) gives the schematic layout of the two dimensional wall jet. Supersonic flow emerges from the nozzle at velocity U_j . For the purpose of our numerical experiment we assume uniform flow at the exit, i.e. viscous effects are neglected. Above the nozzle wall, the external flow has a characteristic speed of U_E . Here too, uniform flow could have been assumed but we choose a boundary layer profile to represent the external flow. The CFD code requires the presence of an external flow; typically U_E needed to be at least 10 percent of U_j for the code to give stable output.

The mean turbulent velocity shown in Fig. (1) is a "self similar" profile that appears downstream of the nozzle (about $50 \times b$). y_m is the location of the velocity maximum. U_m and $y_{1/2}$ scale with downstream distance. For a discussion on

self-similarity see Dash (1986). Here we will describe a typical run for the case where $U_j=1190$ ft/sec and $U_E=300$ ft/sec.

Figs (2) and (3) show the evolution of the streamwise and normal velocities with distance downstream of the nozzle. Figs. (2a) and (3a) respectively, show the u and v velocities at the nozzle exit, and figs. (2i) and (3i) show the velocity profile at a downstream position which is 50 times the nozzle height. In Fig. (2a) note the "top hat" profile at the nozzle exit and the boundary layer profile above the nozzle wall. A combination of figs. (2) and (3) is shown in the velocity vector plot, fig. (4). The y scale has been magnified here to show the otherwise small normal entrainment. Also note that in this figure all arrows are of the same size, hence only the direction of the flow can be determined using this diagram. Fig. (5) shows the Mach number for the flow.

As mentioned earlier, fig. (2a) and (3a) show the velocities in the plane of the nozzle. The top hat u velocity merges with the boundary layer type profile on top of the plate. As the velocity develops with distance downstream, the u and v profiles begin to achieve "self-similar" forms.

So as to perform spatial stability analysis on these profiles, it would be necessary to have much better resolution to ensure that the higher derivatives of the mean velocity can accurately be determined. A certain amount of smoothing, or perhaps model fitting may be required to obtain robust information from the linear stability programs. In addition, a considerable amount of parametric studies are necessary before the profiles obtained from WJET can be considered suitable for stability analysis.

The code contains methodology capable of generating flows for "under expanded" and "over expanded" conditions which would have shock present in the flow field. We were unable to get the code to show shock waves for these conditions.

The generic flow situation in a wall jet, as in any free jet would have shocks embedded in it, hence it is important that further work be done with the code to resolve this problem. To compute the broadband shock associated noise based on the theory of Tam (1987), data from this program would be necessary.

At supersonic velocities, free jet data indicate that compressibility effects can markedly reduce jet growth and mixing. A good data base to isolate the influence of compressibility effects on wall jets is not available, which leads to some uncertainty. Measurements on supersonic wall jets performed in the jet noise laboratory at Langley would greatly facilitate calibration of the turbulence models and better quantify the effect of compressibility. Only when the hydrodynamic data are fairly reliable will the estimation of the noise field be of value.

b) Asymptotic Methods

It has been noted experimentally that in supersonic jets, a small perturbation in the temperature, for example, can have a dramatic effect on the mean velocity profile. For a theory to explain such effects, it should allow for effects of the perturbation on the mean velocity profile to be of $O(1)$. As a matter of fact, in such flows the mean and the perturbation have an almost symbiotic existence, a view conjectured by Malkus (1956) in the context of turbulent flows. A specific example of this type of fully nonlinear interaction has been explicitly displayed by Hall and Lakin (1988) in the context of the development of a Gortler vortex in the growing boundary layer on a curved wall for incompressible flow. Here a coupled set of partial differential equations link the mean flow and the amplitude of the Gortler vortices. An asymptotic analysis is possible for vortices with small wavelengths, while a numerical approach is required when the wavelength is of $O(1)$. It is shown, using this "mean field" approach that the effect of the perturbation on the unperturbed flow can be of $O(1)$.

While the methods used by Hall and Lakin (1988) only deal with the near field instabilities, it is essential in the free shear layer problem that far field pressure estimates be derived. This is particularly relevant as our goal is to determine the far field pressure. It is envisaged that a consistent asymptotic approach developed by Lakin and Reid (1982) for boundary layers can be adapted to give these estimates. This approach, which has been successful for incompressible flows, provides fully uniform asymptotic solutions throughout the near and far field of the flow.

As in Hall and Lakin (1988) it is conjectured that the disturbance evolves nonlinearly and is bounded by mixing layers that confine its spreading in the normal direction.

Within these layers the convective terms balance diffusive terms in the perturbation equations, with a $O(\epsilon)$ density perturbation causing an $O(1)$ effect on the mean velocity. The mean velocity and the perturbation quantities are connected by coupled equations. The perturbation equation is of the Orr-Sommerfeld type, with the unknown mean flow appearing in the coefficients. The nonhomogeneous equation for the mean, with known coefficients is forced by terms involving the perturbations quantities on the right hand side. The scalings for the flow and spatial variables are to be determined. By using these scales we expect to obtain a matching of the perturbation quantities within the mixing layer and their asymptotic behavior at infinity, thereby arriving at a uniform asymptotic expansion for these quantities.

The uniform expansions obtained help us avoid the drawback of the conventional method of far field estimation used by Tam and Morris (1980). In their method a self similar profile for the mean flow is assumed and quasi-parallel stability theory is used to investigate the evolution of a linearly unstable perturbation. This method leads to a non-uniformity in their expansions.

No significant results have been obtained in this area but it is expected that this problem will be pursued by Dr. W. Lakin beyond the completion of this grant. Results from this effort will be reported to the Aeroacoustics Branch.

c) Stability of Flows with Transverse Curvature

In recent years there have been a number of experimental and theoretical investigations of supersonic flow past sharp and blunted cones. Despite these studies, there remain a number of unresolved fundamental issues about the stability and transition of these flows.

Recent work, Malik (1989) indicates the importance of transverse curvature effects, and confirms previous asymptotic results showing stabilization with curvature for axisymmetric first and second modes, and destabilizing for oblique

first mode disturbances. In contrast, the work of Macaraeg and Daudpota (1990) shows that axisymmetric (2D modes) may be destabilized by curvature, depending on the value of the Reynolds number, Re . Oblique first modes are, however, found to be destabilized, confirming previous theoretical studies, regardless of Re for the cases studied. Oblique second modes are found to be stabilized regardless of Re for the conditions studied. It is also shown that although curvature effects may be important for Mach numbers below 5; this significance sharply decreases at higher Mach number. Thus the above studies are not necessarily contradictory. Generalization about the effect of curvature may only be attempted after further theoretical and numerical analysis.

For studying the effect of transverse curvature on the linear stability of a flow field, we consider a similarity boundary layer profile of a flow along the axis of an infinitely long cylinder. By considering cylinders of different radii, the effect of curvature can be determined.

Another aspect of this study is the flow over a cone which has a farfield shock associated with it. The effect of the placement of the farfield boundary, the resolution and the curvature on the stability characteristics are investigated. In previous studies insufficient attention has been paid to the problem of proper boundary conditions. The current study is a preliminary look at these effects.

The effect of curvature on the temporal stability of compressible adiabatic flow has been considered with special reference to axisymmetric first, second and supersonic modes. Oblique modes are considered briefly. In addition to investigating different curvatures, a range of Re and Mach numbers is considered. Results for low curvatures approach those obtainable on a flat plate with the same flow conditions.

First axisymmetric modes are found to be stabilized by curvature at low Reynolds number. A destabilization with increasing curvature, however, is seen

as Reynolds number is increased at constant Mach number. The Reynolds number at which this trend reversal occurs is higher at lower Mach numbers. In other words, if Reynolds number is held constant and Mach number is increased, it is found that curvature will tend to destabilize axisymmetric first modes. In the planar limit, growth rate decreases with Mach number for a fixed Re , in agreement with previous Cartesian results.

Unlike the first modes, supersonic modes in the planar limit do not have growth rates that decrease monotonically with Mach number. Another feature distinguishing these modes is that the supersonic ones tend to be destabilized with curvature as Mach number is lowered at constant Re .

The dominance of the first mode at low streamwise wavenumber ($\alpha=0.2$) is no longer true at $M_\infty=2$. At these latter conditions a supersonic mode, oscillatory to the farfield, has the highest growth rate, although the first mode is still present.

The second mode is shown to be stabilized with curvature, a finding that has been reported by Malik (1989). However, if extremely high values of Re is used, another inviscid mode appears that is destabilized with curvature. Due to the very large value of Re , it is doubtful whether this mode is of practical significance.

The oblique first mode studied behaves in a manner in keeping with that suggested by previous numerical studies: destabilization with increasing curvature. For the case studied ($M_\infty=1.414$) no trend reversal with Re is noted. In addition, the oblique second mode is seen to be destabilized at all Re , for the case studied ($M_\infty=6.8$).

The stability of flow on a blunt body has been mainly conducted to see the effect of imposing different far field boundary conditions on a flow bounded by a shock. For the case studied it is found that Neumann conditions give rise to unstable disturbances, which decay to a nonzero constant. Dirichlet conditions represent unsuccessful disturbance boundary conditions at a shock.

Experiments have shown the presence of unstable modes at wavelengths higher than those predicted by past linear stability results. Such modes are found to be unstable in the present study but at a higher Re than those in the experiments. These modes have a highly peaked behavior in the critical layer, and their structure near the wall suggests an inviscid character.

d) Supersonic Nozzle Analysis

One of objective of jet noise research is to design and analyze supersonic nozzles, their flow and acoustics. So as to obtain shock-free flows, it is generally necessary to operate the nozzle at conditions somewhat different from those predicted by inviscid theory. It is not clear whether there is one dominant factor causing this discrepancy. It was suspected that perhaps the boundary layer in the subsonic part of the nozzle was significantly thick to make the inviscid predictions invalid.

Korte (1989 and 1990) has shown that a parabolized Navier Stokes (PNS) solution of supersonic flow is very efficient for computing viscous flows in comparison with a full Navier Stokes calculation. In nozzle flows at high Mach numbers, Navier Stokes and PNS solution show very significant boundary layers.

It was therefore decided to analyze a shock-free Mach 2 nozzle using the PNS code. In the mid 1970's Seiner (1990) designed and constructed a Mach 2 nozzle using an inviscid method of characteristics procedure. Experiments were performed on the nozzle which had pressure taps at different circumferential and streamwise positions along the walls. The pressure was recorded for shock free flow conditions.

The Mach 2 nozzle was operated at a pressure of 108.6 psia, with atmospheric pressure equal 14.5 psia and temperature of 74°F. Three circumferential pressure taps were placed at each of the 5 streamwise stations. Fig. (6) shows the shape of the nozzle's supersonic section.

Figs. (7) and (8) show the predicted exit and the centerline Mach numbers for this nozzle. It can be seen that there is a very small boundary layer effect. The code was then run in the inviscid mode giving the exit profile shown in Fig. (9). A comparison of wall pressures between the viscous and inviscid cases is given in

Fig. (10). This shows that there is only a small difference (approx. 0.5 psia) at the exit between these two cases.

Finally a comparison was made between the computed wall pressures and the experimentally determined values. Excellent agreement was found as is seen from Fig. (11). In this run the program was input the experimentally determined starting value for pressure at the nozzle. The circumferential values are p_1 , p_2 , and p_3 . When the theoretical value at the throat nozzle is input as the starting value for the program, computed wall pressure values are consistently above the measured. This however is to be expected since the starting theoretical value input to program is higher than what was measured at the nozzle. The result can be seen in Fig. (12).

Our preliminary exploration suggests that the PNS code (with slight modifications so that it can accept the position of the sonic line as the starting data) can be a useful tool for analyzing nozzles without the need for a Navier-Stokes solution. The code could also be used (again with some modification) for non-axisymmetric flows such as in elliptic and rectangular nozzles.

Conclusions

In two areas considered above, which are of immediate interest to the Aeroacoustics Branch, i. e. wall jet flows and nozzle flows, it is recommended that considerable more effort be expended both on computational and experimental aspects.

The supersonic wall jet has turbulence model and compressibility effects problems associated with it. Experimental results would greatly facilitate the calibration of turbulence models for such flows in addition to providing information about how compressibility influences the interaction of the free shear layer and the boundary layer flow.

It is suggested that pressure taps be installed in some non-axisymmetric nozzles and the experimental results compared with the 3D version of the PNS code. Further, a Navier-Stokes code should be used to get the correct upstream flow. This would provide an accurate location of the sonic line which can then be input to the PNS code.

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Figures

- Fig. 1. The plane wall jet: Configuration.
- Fig. 2. Variation in streamwise velocity with distance from the nozzle.
Nozzle velocity = 1190 ft/sec. $U_E = 334$ ft/sec. Nozzle height = 0.003 ft.
- Fig. 3. Variation in normal velocity with distance from the nozzle.
Parameters identical to those in Fig. 2.
- Fig. 4. Velocity vector plot of the same flow as in Figs. (2) and (3).
- Fig. 5. Mach number plot for wall jet.
- Fig. 6. Mach 2.0 nozzle contour.
- Fig. 7. Exit Mach number profile.
- Fig. 8. Centerline Mach number variation from nozzle throat to the exit.
- Fig. 9. Exit Mach number profile, using Euler equations.
- Fig. 10. Comparison of wall pressure for viscous and inviscid flow equations.
- Fig. 11. Comparison of experimental and numerical values of wall pressure.
Pressure at throat for the PNS code taken as the experimental measured value.
- Fig. 12. Comparison of experimental and numerical values wall pressure.
Pressure at throat for PNS code taken as the theoretical computed (inviscid calculation) value.

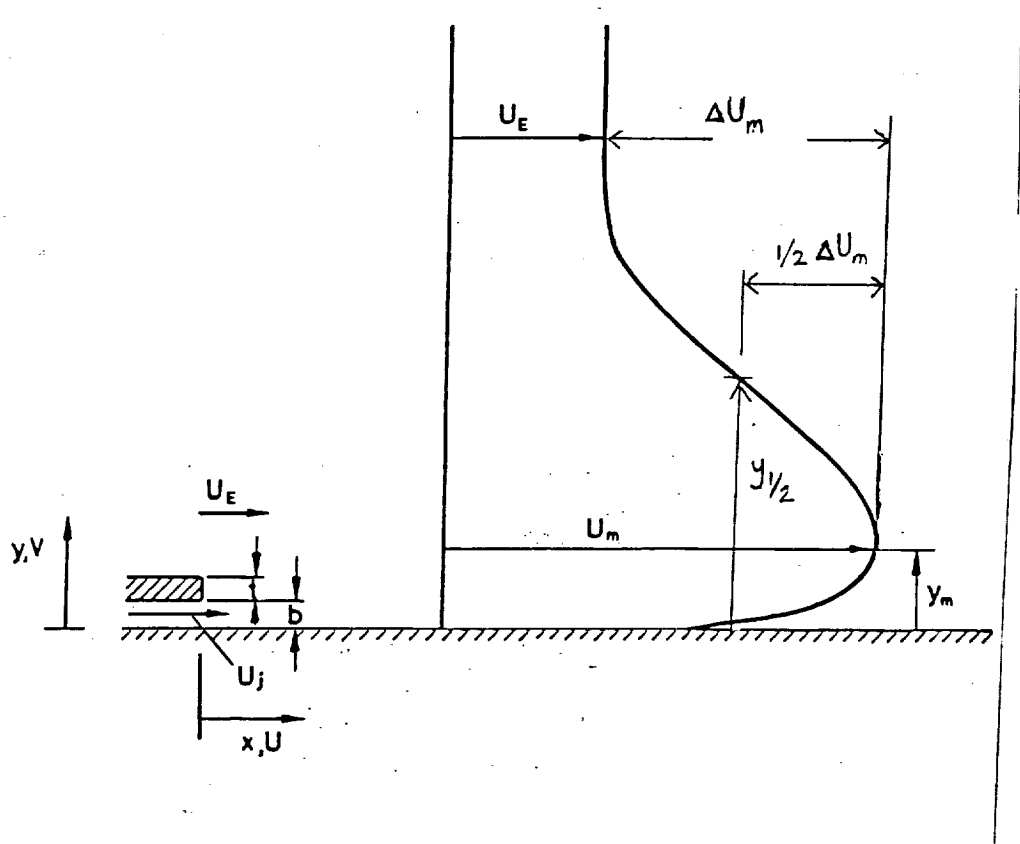


Fig (1)

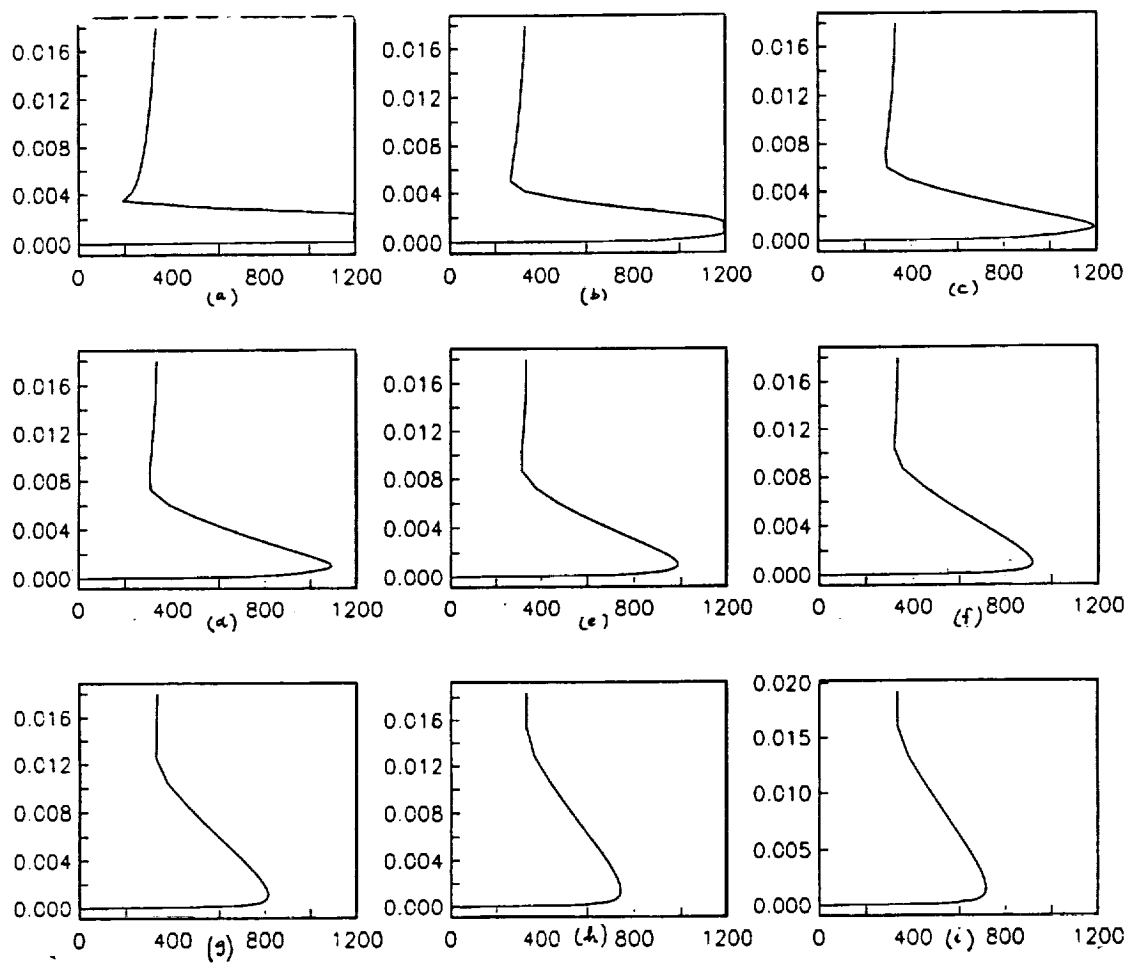


Fig (2)

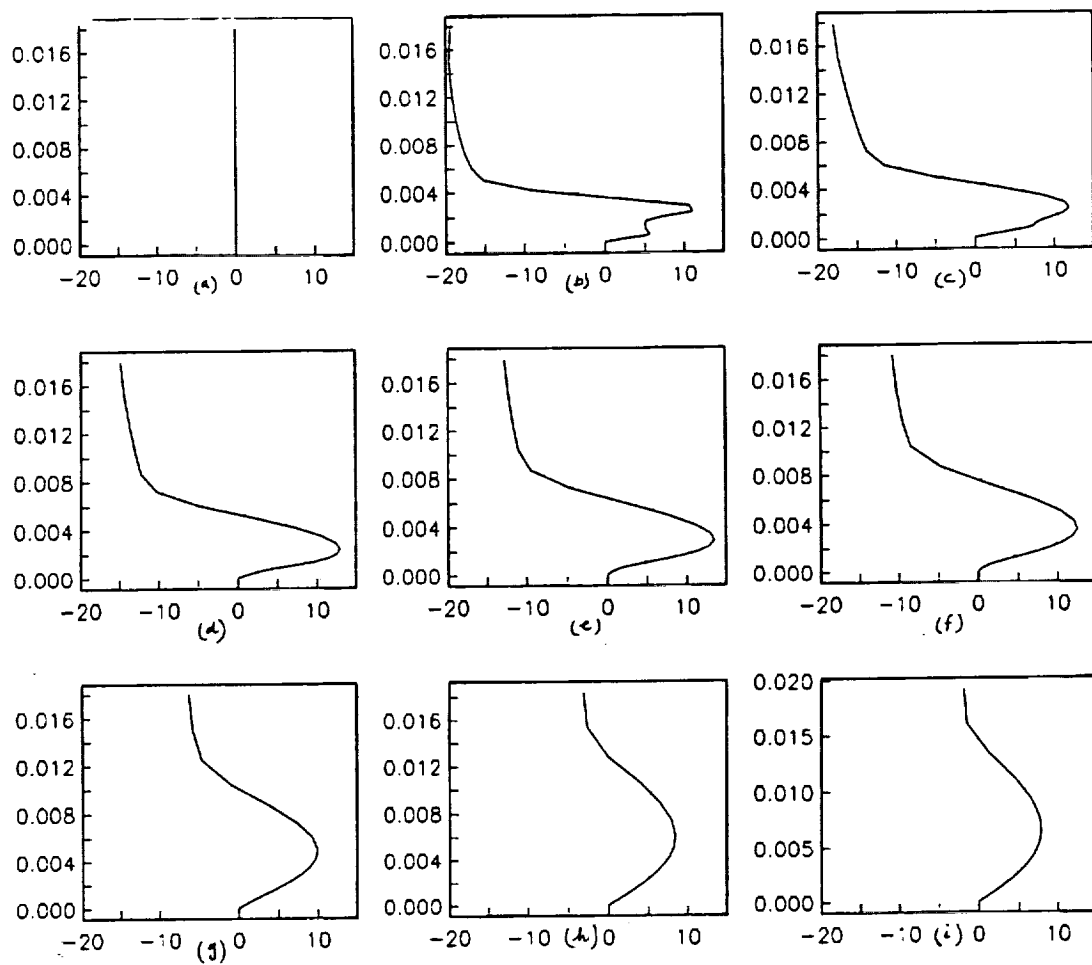


Fig (3)

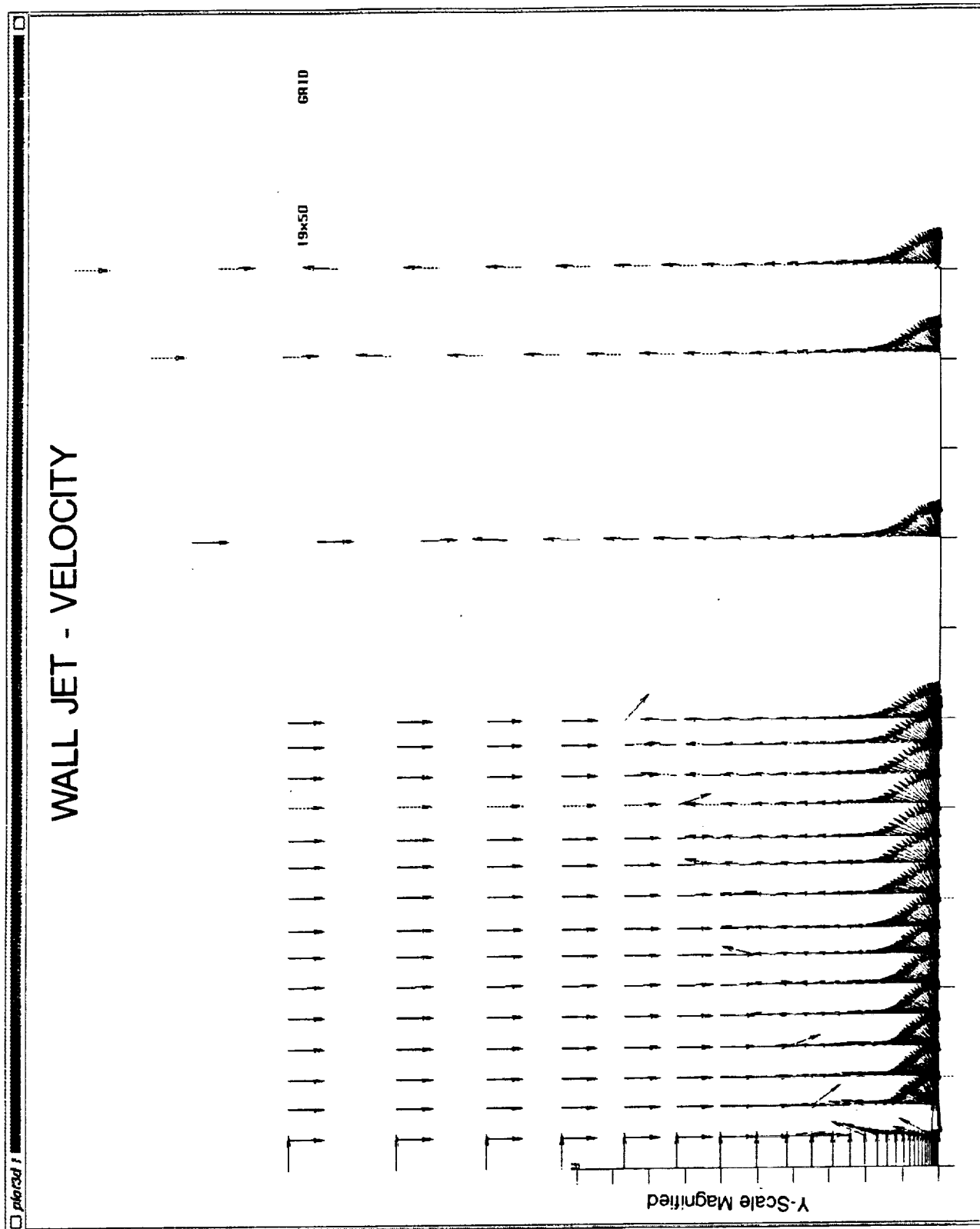


Fig (4)

Mach 2.0 Nozzle Contour

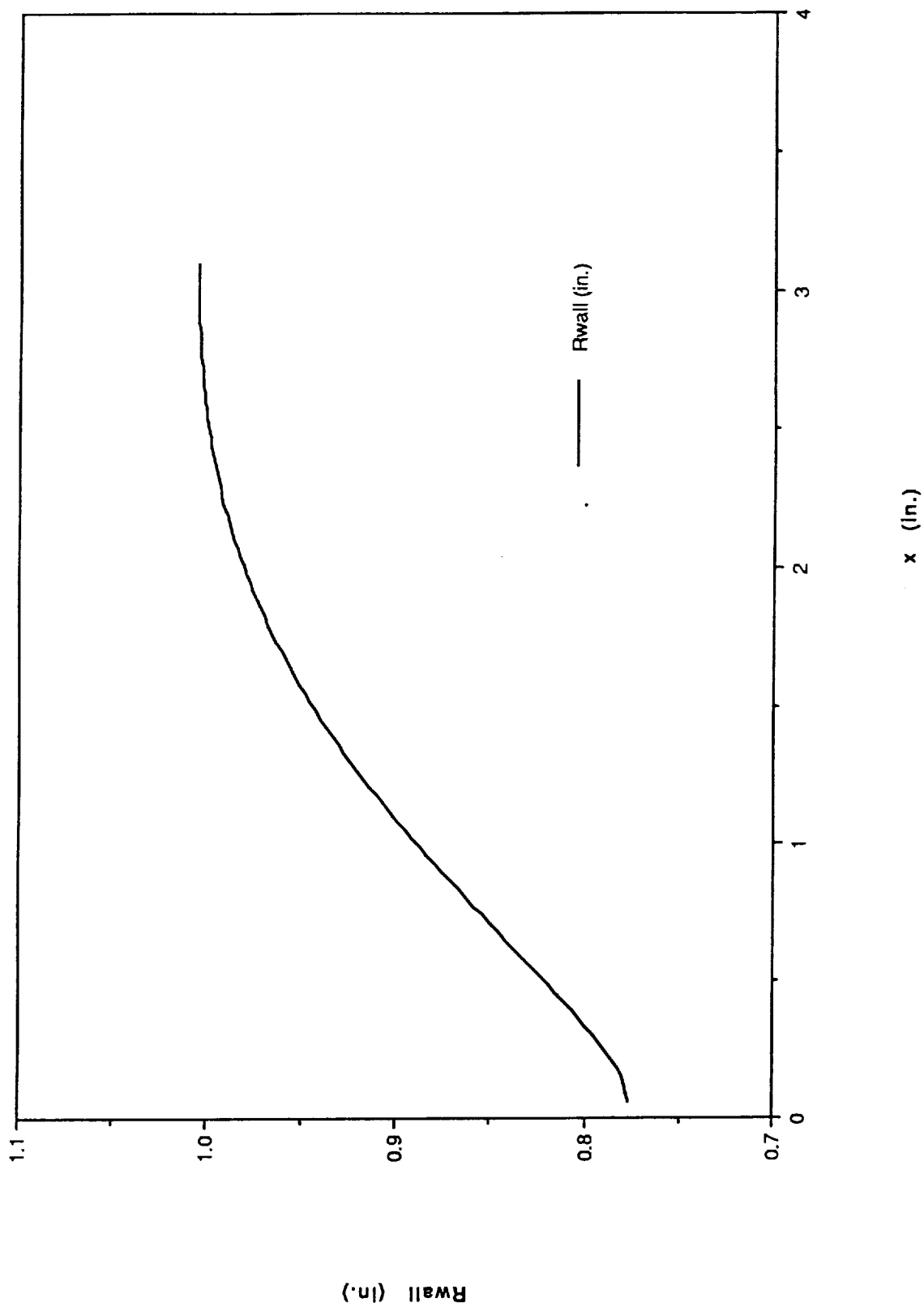


Fig (6)

Exit Mach Profile

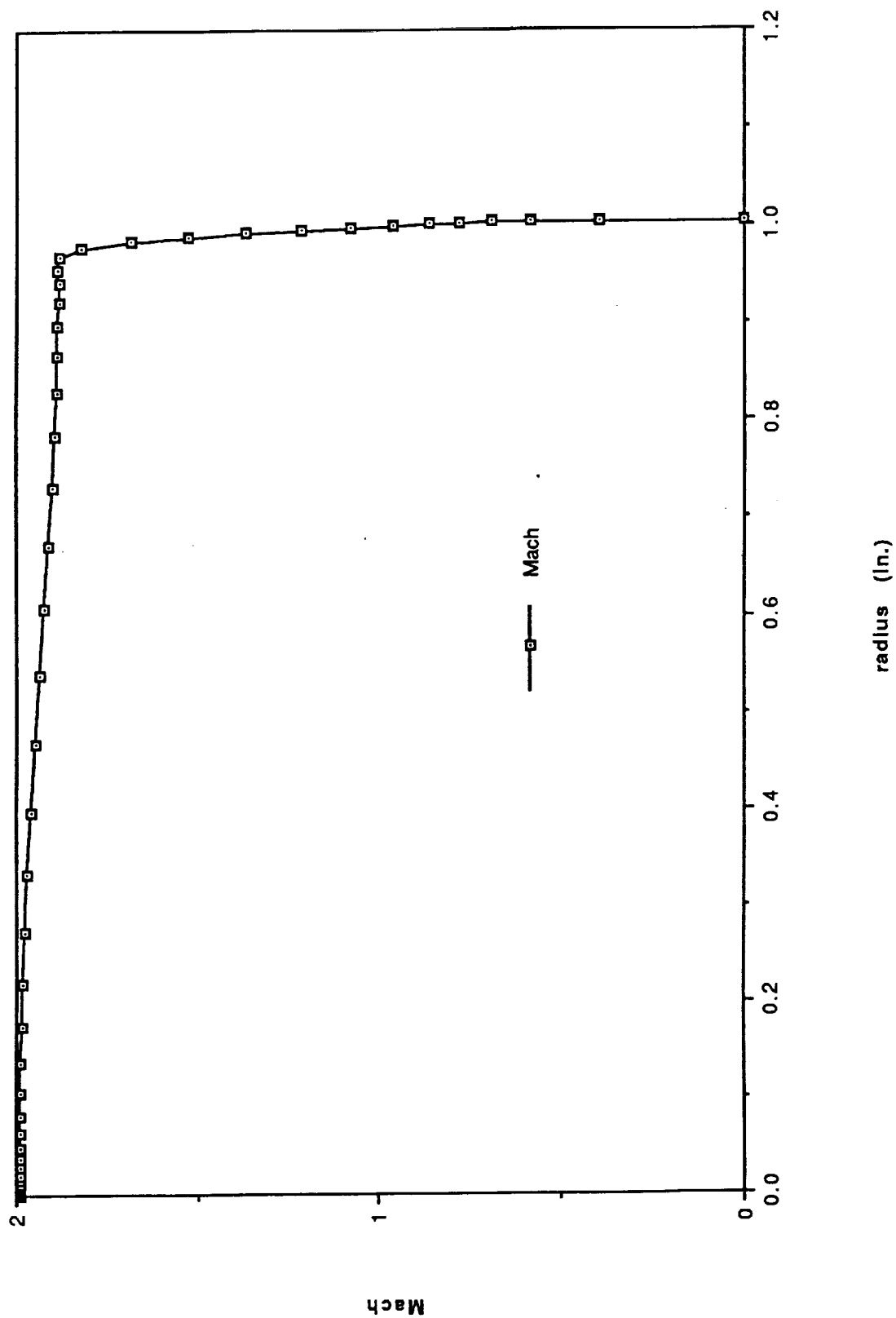
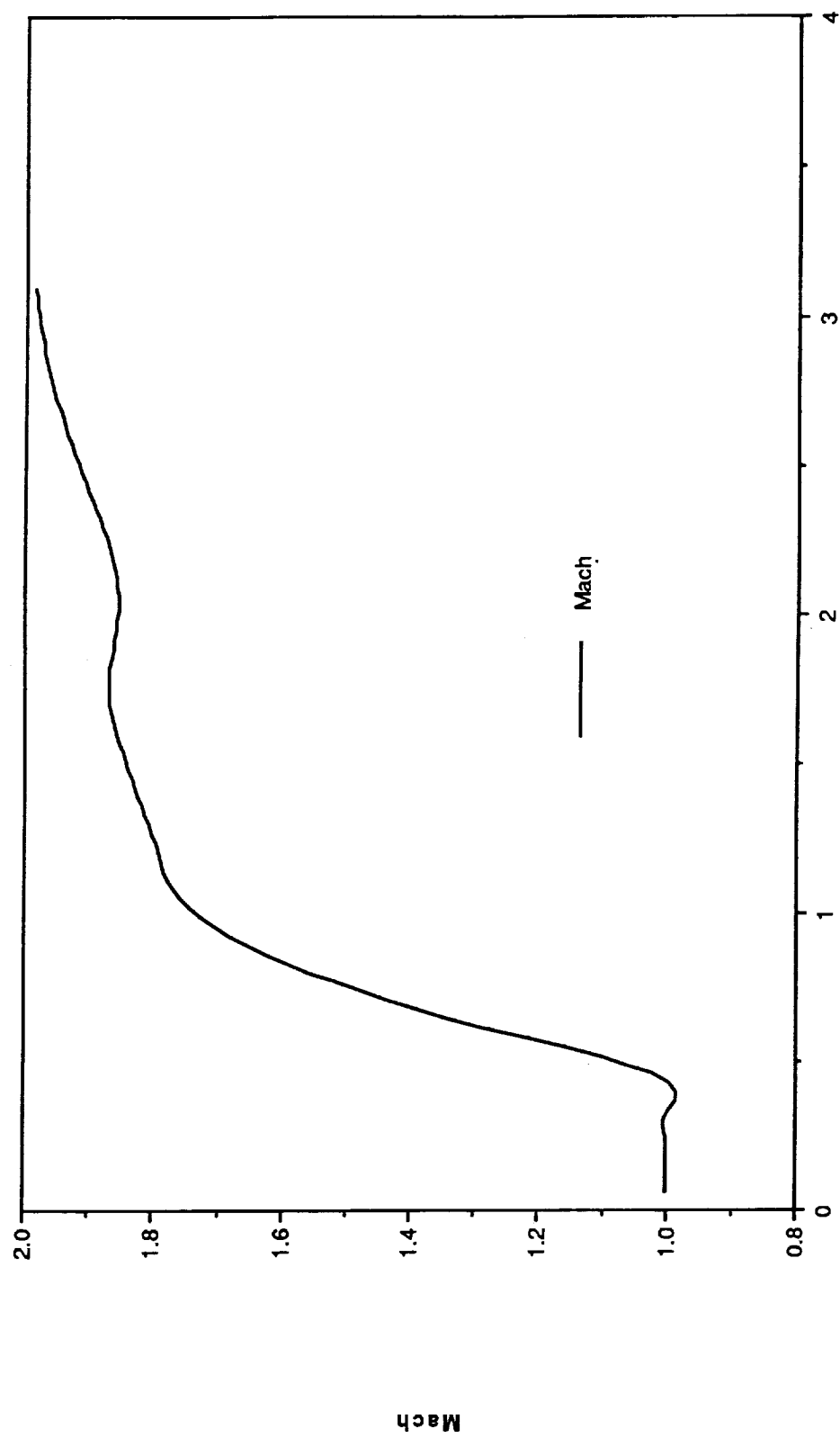


Fig (7)

Centerline Mach Distribution



x (in.)

Fig (8)

Inviscid Mach Profile

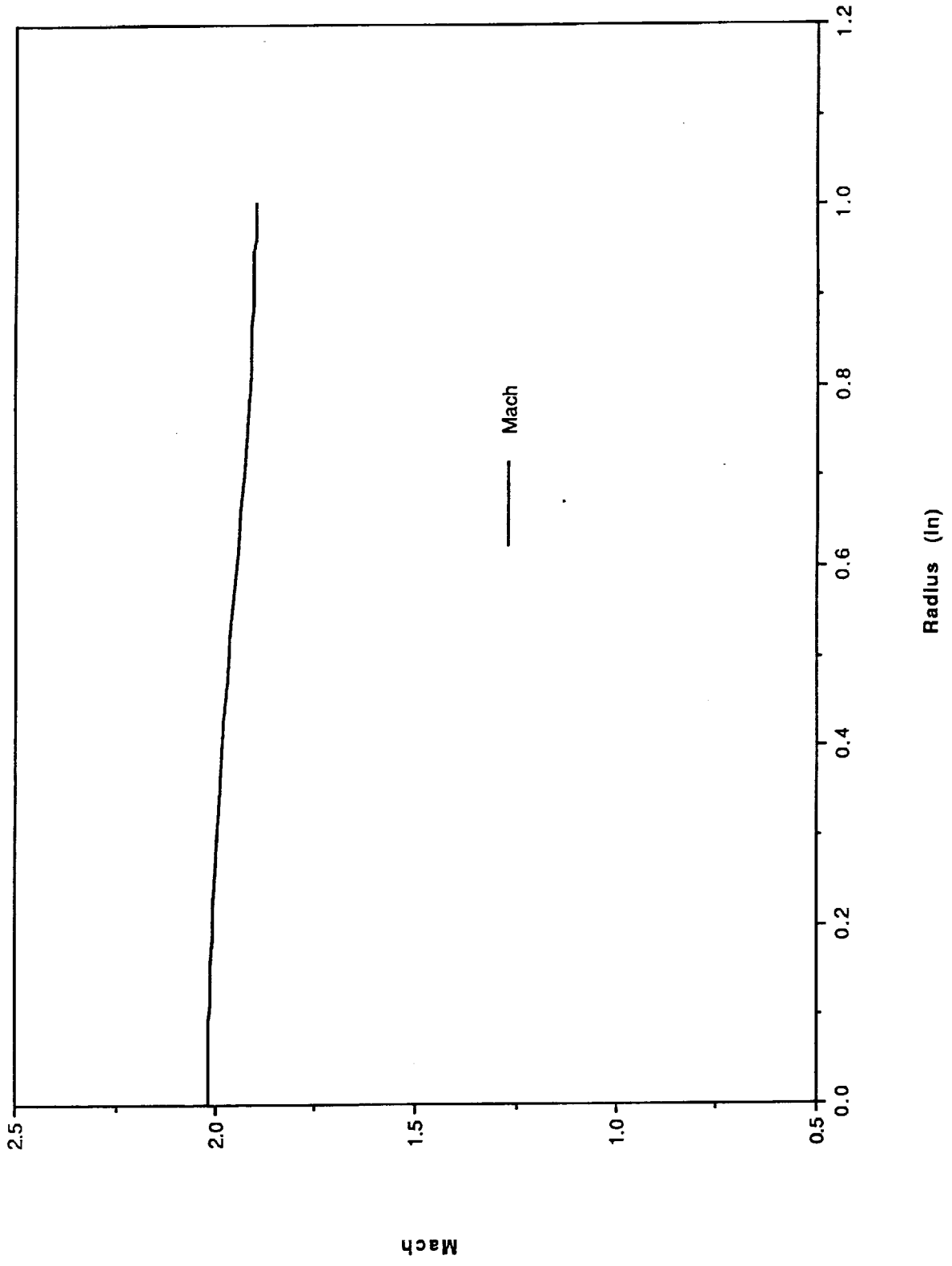
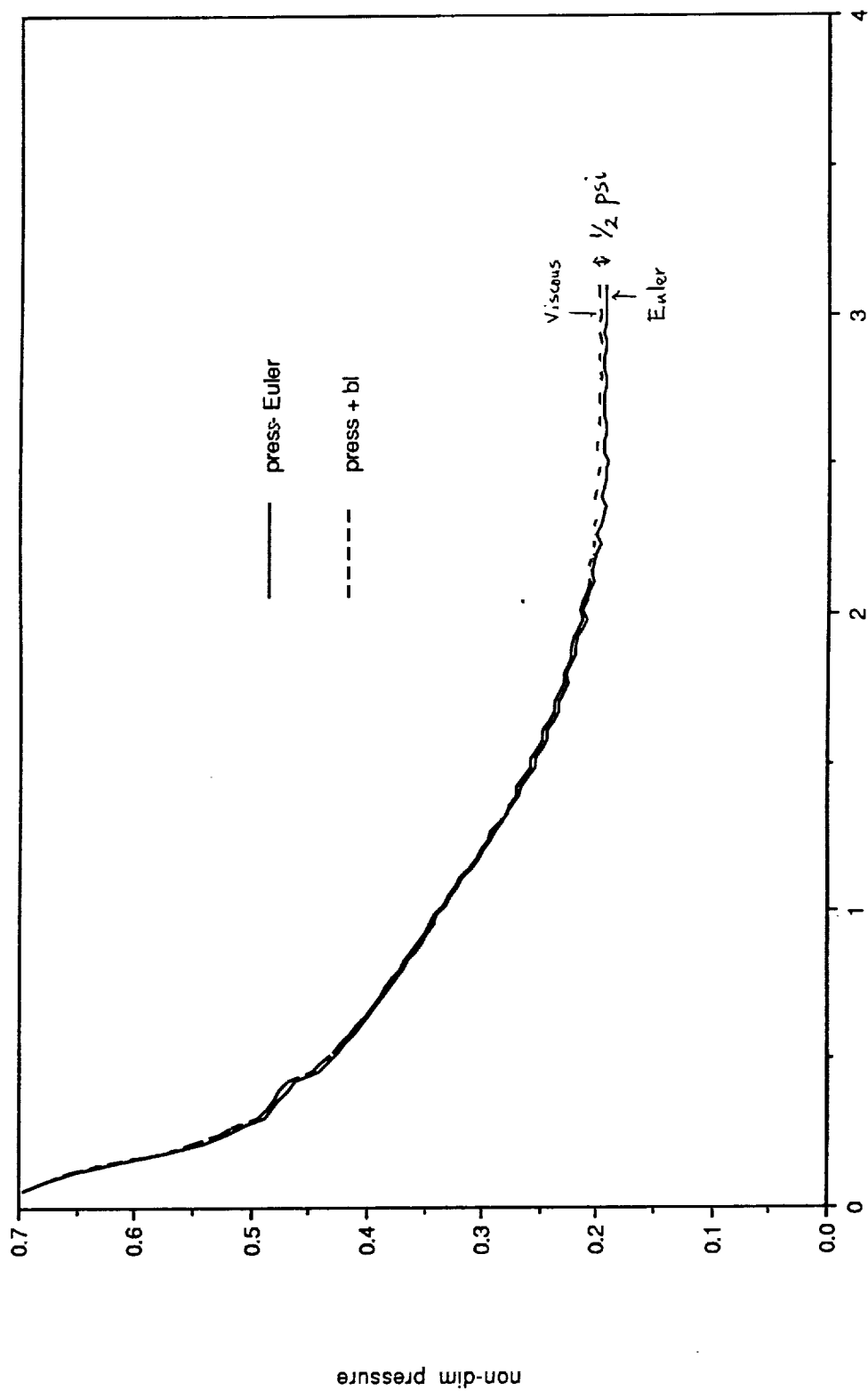


Fig (9)

Comparison of Wall Pressure with and without Boundary Layer



$x(\ln.)$

Fig (10)

Axial Wall Pressure Distribution

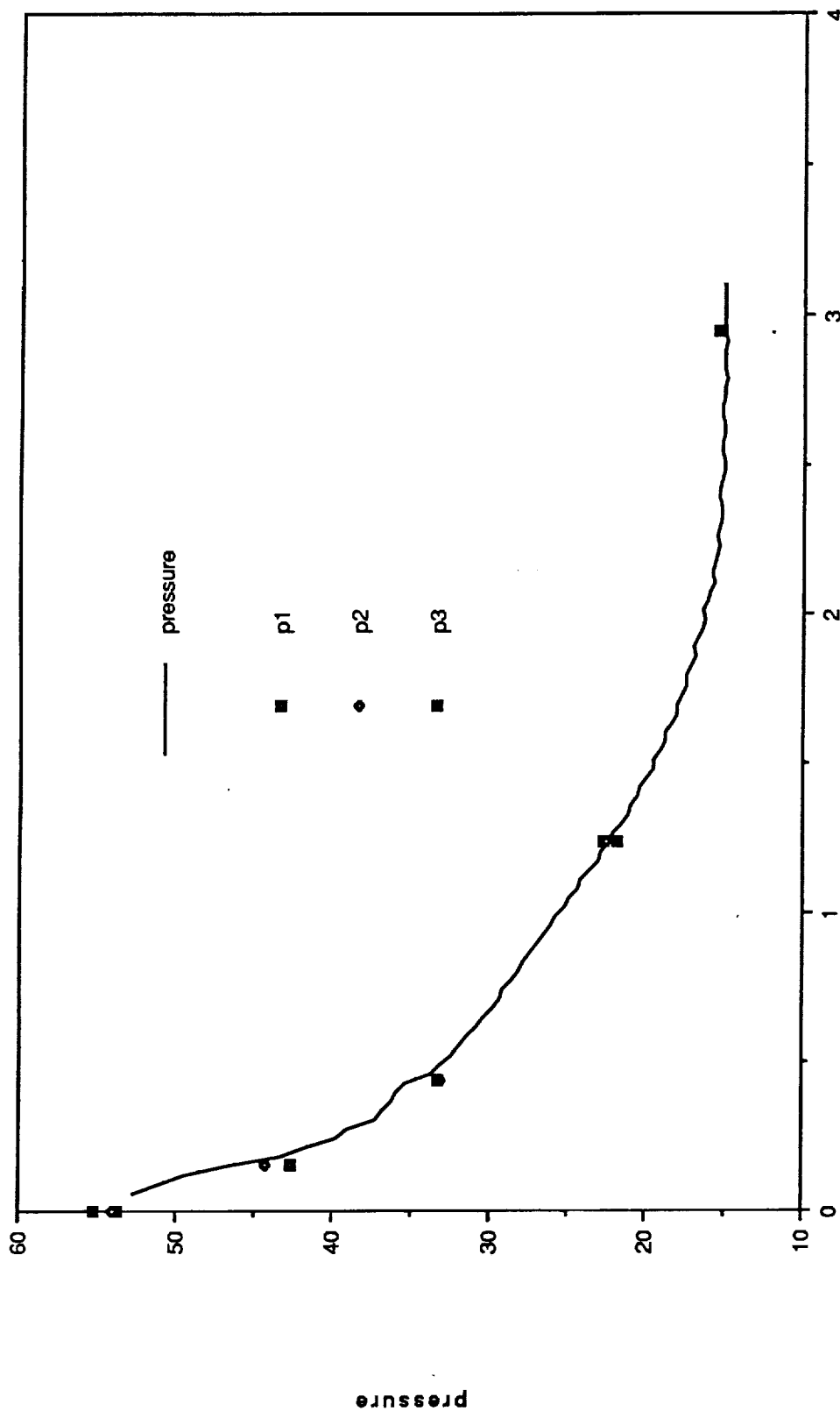
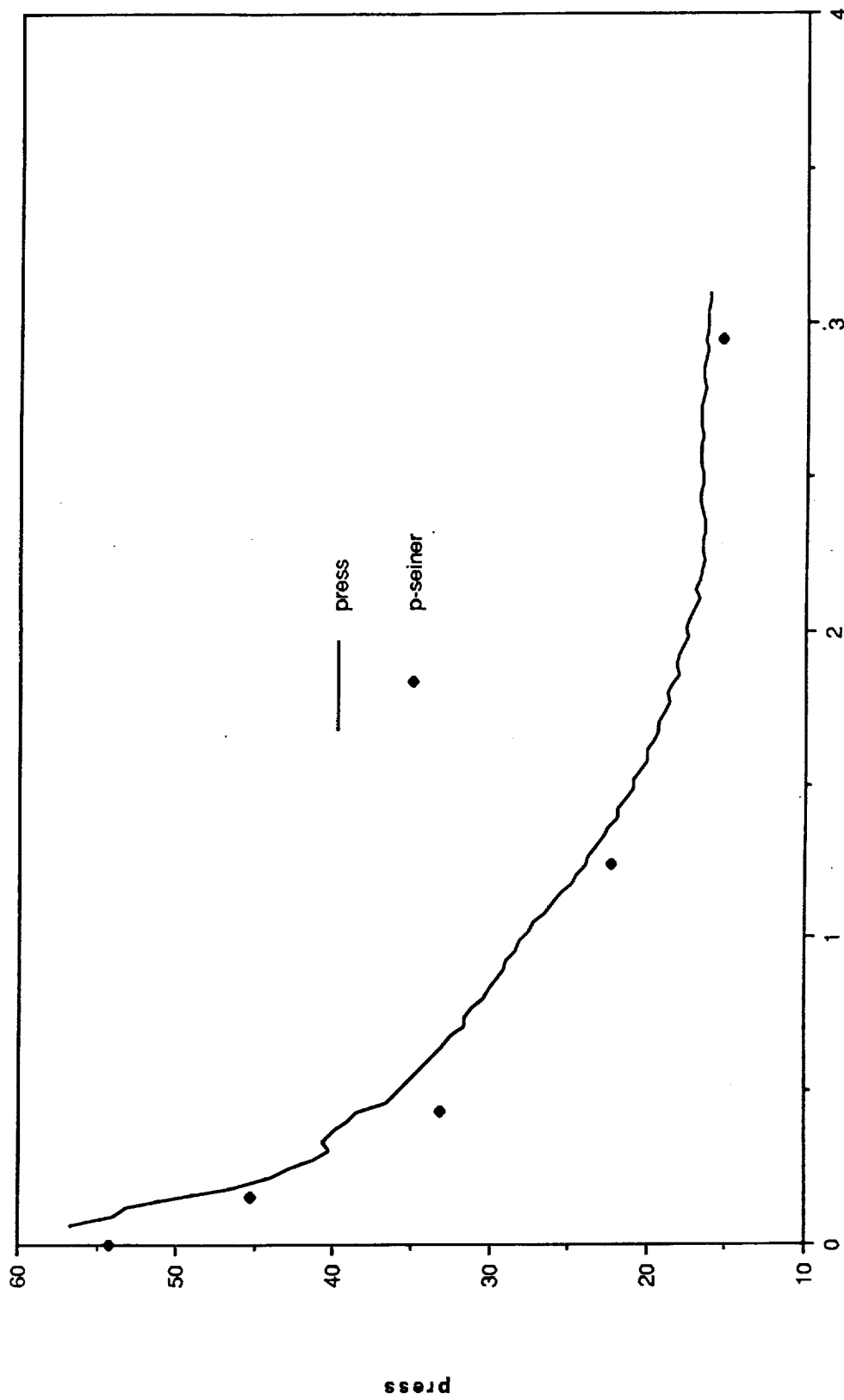


Fig (11)

Mach 2.0 Wall Pressures



x (in.)

Fig (12)